Science and Technology

Stochastic Interacting System

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Background of Research

Natural world and human society are filled with phenomena that can be described by a stochastic process such as movement of fine particles floating on the water or fluctuations of stock prices. In case a system is made up of a number of component elements, the mutual interactions among the elements are playing an important role in many cases. Iron turning to magnet or a traffic congestion occurring on a highway is a few examples of that.

By means of analysis on a simple model system which carries essential characteristics of such phenomena, we explored the universal mathematical structure therein. The obtained knowledge from there is expected to be useful for devising an analytic approach to more complicated phenomena.

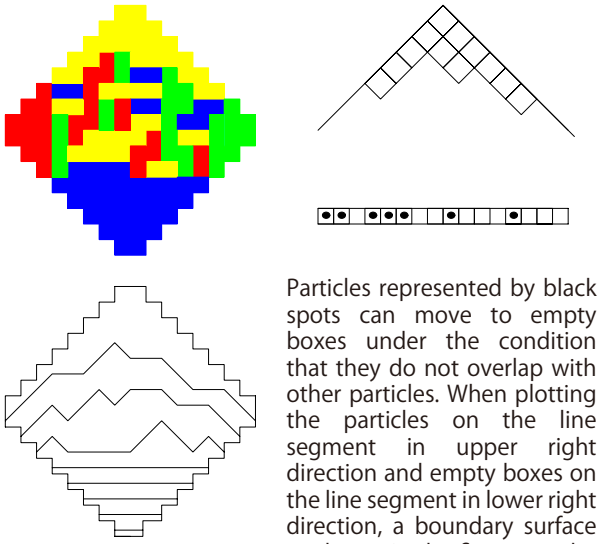
Achievements of Research

A model where a number of particles are randomly walking on a one-dimensional lattice under the condition that they do not overlap with one another is called an asymmetric exclusion process. This process can be regarded as a simple model of traffic flow or as related to various phenomena such as boundary growth or combinational optimization. (See the figure) It is also known that the interactions among the particles can exhibit interesting situations.

Recently I am studying the characteristics of fluctuations in such a stochastic process. As an example, focusing on one particle in this process, I conducted a precise calculation of the correlation between the times where the position fluctuations of that particle are different. As a result, it was found out that the correlation can be described with a Fredholm determinant using a function called Airy function. This is a fluctuation model with a new universality which has not been recognized with a system of only one particle, and discussion is under way about its relation with random matrix theory. Although it is a result of research on a special system, it is very rare to be able to obtain such detailed information for an interacting system, and that can provide a useful reference for consideration of more universal characteristics of a stochastic interacting system in the future.

Prospect of Research

The studies of stochastic interacting systems have been greatly advancing in recent years. The understanding of such systems has been deepened and considerably specific results have been obtained for some of the high quality models, showing signs of algebraic structures behind them. Moreover, the studies of such systems are closely related to the physical theory of non-equilibrium statistical mechanics, and therefore further development is expected while interacting with other various study fields in the future.



Particles represented by black spots can move to empty boxes under the condition that they do not overlap with other particles. When plotting the particles on the line segment in upper right direction and empty boxes on the line segment in lower right direction, a boundary surface as shown in the figure can be derived.

Fig.: Asymmetric exclusion process is related to boundary growth (above on the right) or tiling of Aztec diamond (above on the left) and can be solved by using the relation with non-crossing walk (below on the left).